



EE461 Midterm:

NAME: Ryan Oide

STUDENT NO.: 913142

Date: Monday, October 20, 2003

Time = 2 hours

Text Books and Notes Only

Absolutely no worked examples or solved problems

NB: Draw a box around your final answer.

Part A

1. 5

2. 5

3. 5

4. 5

SUBTOT 20

1. A causal system, i.e. $h(n) = 0$ for $n < 0$, has impulse response $h(n) = (-0.5)^n$ (obviously n is an integer from the interval $(-\infty, \infty)$).

(3)

(a) What is the frequency response of the system?

(2)

(b) If the input to the system is $2.0\cos(0.25\pi n + \pi/4)$, what is the output?

(a) $h(n) = (-0.5)^n$

$$H(e^{j\omega}) = \sum_{n=0}^{\infty} h(n) e^{-j\omega n} = \sum_{n=0}^{\infty} (-0.5 e^{-j\omega})^n = \frac{1 - (-0.5 e^{-j\omega})^{\infty}}{1 - (-0.5 e^{-j\omega})}$$

$$H(e^{j\omega}) = \frac{1}{1 + 0.5 e^{-j\omega}} \quad \checkmark$$

(b) $H(e^{j\frac{\pi}{4}}) = \frac{1}{1 + 0.5 e^{-j\frac{\pi}{4}}} = 0.7149 e^{-j0.255}$

$$y(n) = 0.7149 e^{-j0.255} \cdot 2.0 \cos\left(\frac{\pi}{4}n + \frac{\pi}{4}\right)$$

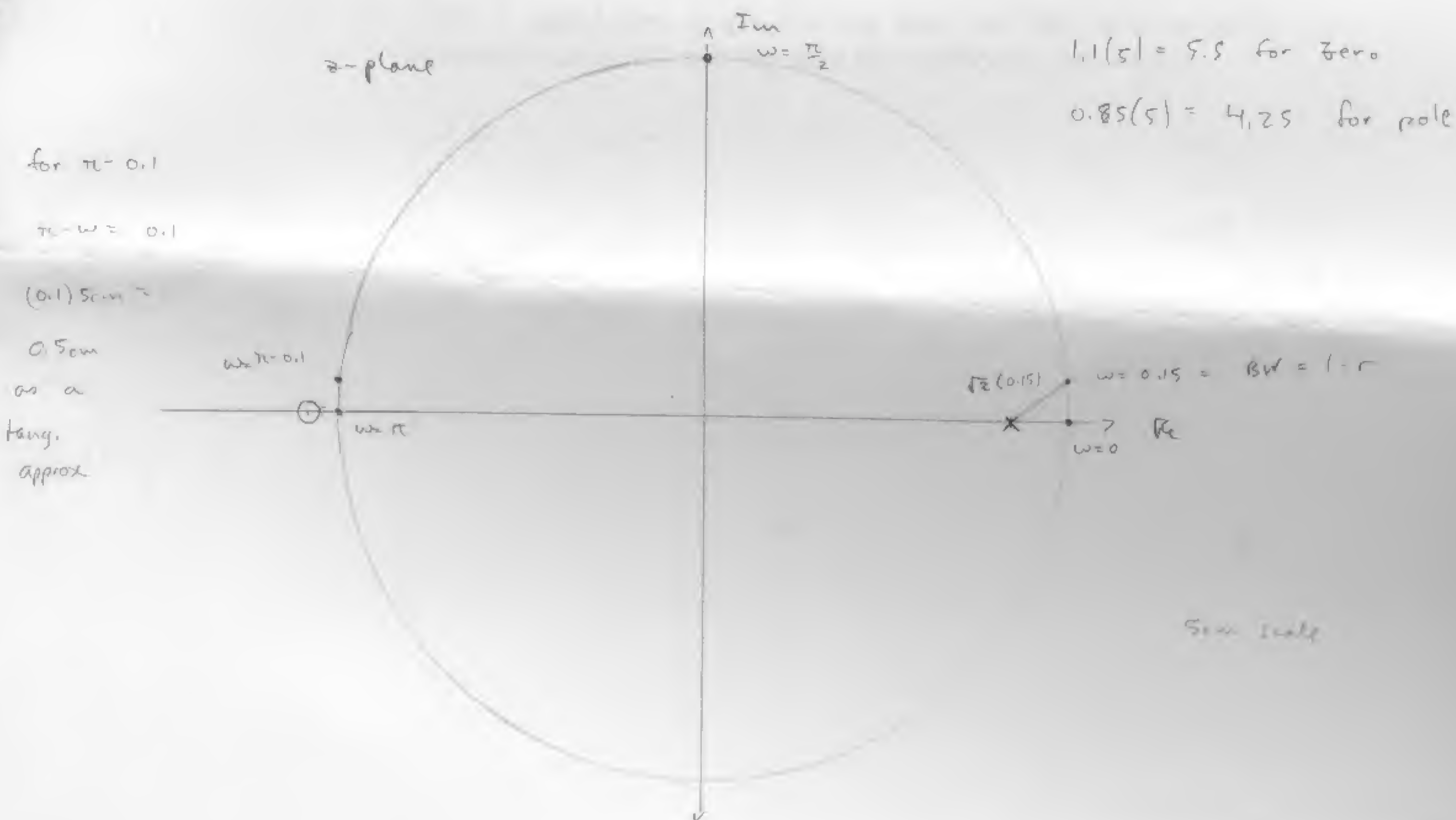
$$= 1.43 \cos\left(\frac{\pi}{4}n + \frac{\pi}{4} - 0.255\right)$$

$$y(n) = 1.43 \cos\left(\frac{\pi}{4}n + 0.5304\right)$$

1.04

(5)

2. Using graphical methods, find the magnitude response of a system with one zero at $z = 1.1e^{j\pi}$ and one pole at $z = 0.85e^{j0}$ for frequencies $\omega = 0, 0.15, \pi/2, \pi - 0.1$ and π radians per sample. Assume $b_0 = 1$.



$$|H(e^{j\omega})| = \frac{b_0 |z - 1.1e^{j\pi}|}{|z - 0.85e^{j0}|}$$

$$\text{at } \omega = 0 \quad |H(e^{j0})| = \frac{1.1 + 1}{0.15} = 14$$

$$\omega = 0.15 \quad |H(e^{j0.15})| = \frac{10.5 \text{ cm} / 5 \text{ cm}}{\sqrt{2}(0.15)} = 9.90$$

$$\omega = \frac{\pi}{2} \quad |H(e^{j\frac{\pi}{2}})| = \frac{7.4 \text{ cm} / 5 \text{ cm}}{6.6 \text{ cm} / 5 \text{ cm}} = 1.12$$

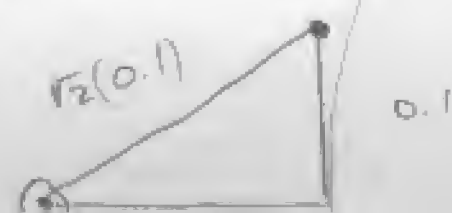
$$\omega = \pi - 0.1 \quad |H(e^{j(\pi - 0.1)})| = \frac{\sqrt{2}(0.1)}{9.3 \text{ cm} / 5 \text{ cm}} = 0.076$$

$$\omega = \pi \quad |H(e^{j\pi})| = \frac{0.1}{0.85 + 1} = 0.054$$

For $\omega = \pi - 0.1$

unit circle

Tang. approx



$$|e^{j(\pi - 0.1)} - 1.1e^{j\pi}| \approx \sqrt{2}(0.1)$$



z plane

0.85/5 = 4.25 for pole

for $\pi = 0.1$

$\pi - \omega = 0.1$

$(0.1) 5cm =$

0.5cm

as a

tang.

approx

$\omega = \pi - 0.1$

$\omega = \pi$

$\sqrt{2}(0.15)$

$\omega = 0.15 = BW = 1 - r$

$\omega = 0$ Re

Same as previous page

5cm scale

$$|H(e^{j\omega})| = \frac{|z - 1.1e^{j\pi}|}{|z - 0.85e^{j0}|}$$

$$\text{at } \omega = 0 \quad |H(e^{j0})| = \frac{1.1 + 1}{0.15} = 14$$

$$\omega = 0.15 \quad |H(e^{j0.15})| = \frac{10.5cm/5cm}{\sqrt{2}(0.15)} = 9.90$$

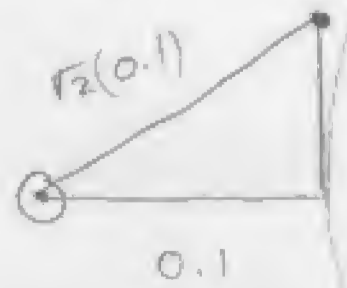
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$$\omega = \frac{\pi}{2} \quad |H(e^{j\frac{\pi}{2}})| = \frac{7.4cm/5cm}{6.6cm/5cm} = 1.12$$

$$\omega = \pi - 0.1 \quad |H(e^{j(\pi-0.1)})| = \frac{\sqrt{2}(0.1)}{9.3cm/5cm} = 0.076$$

$$\omega = \pi \quad |H(e^{j\pi})| = \frac{0.1}{0.85 + 1} = 0.054$$

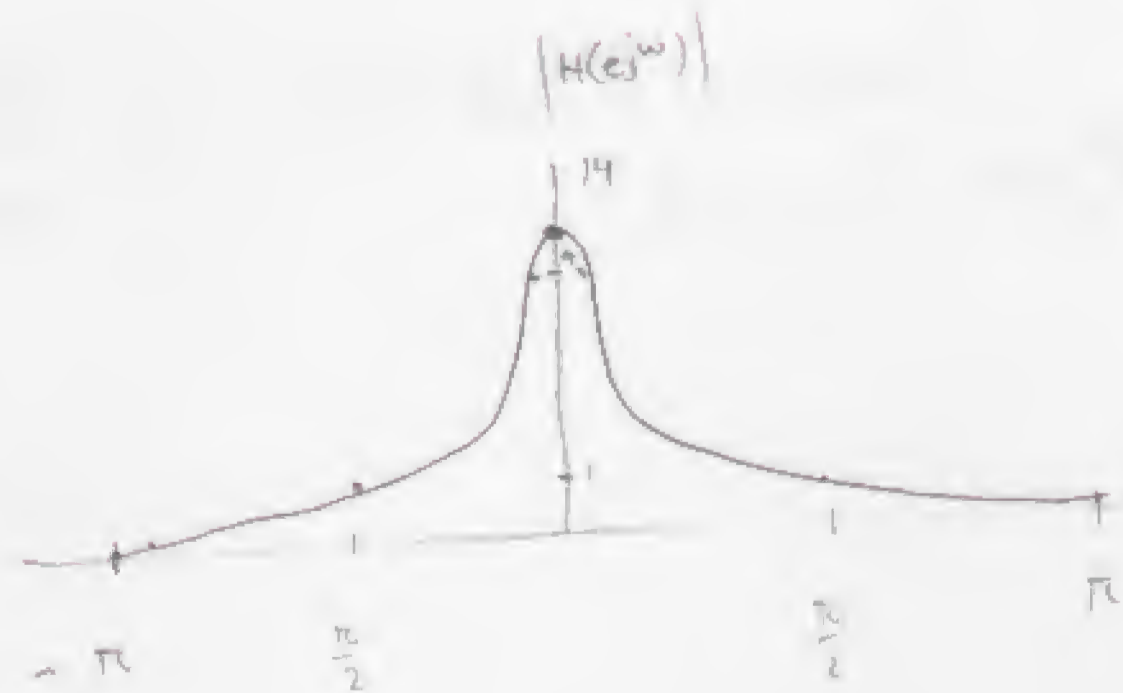
For $\omega = \pi - 0.1$



unit circle

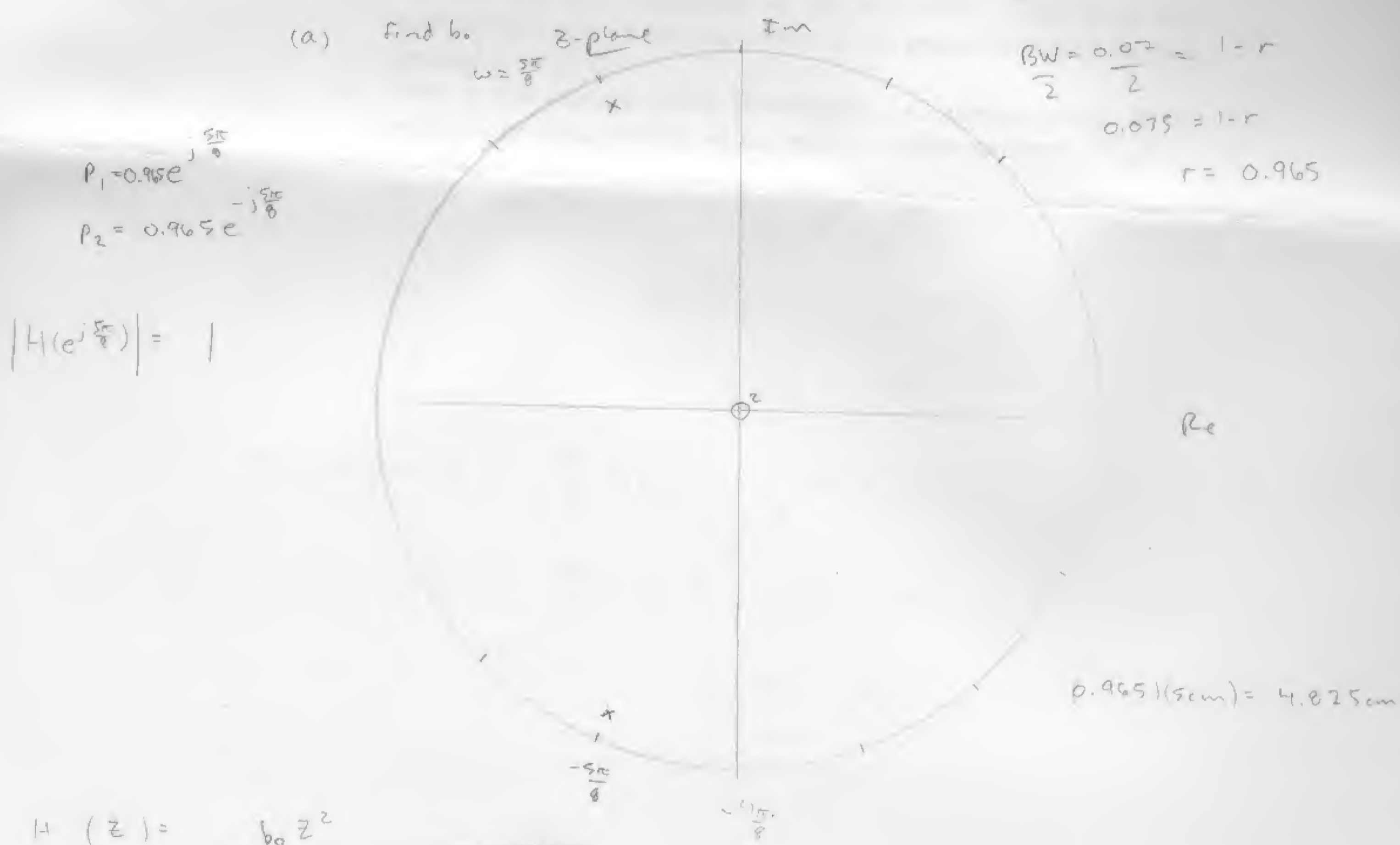
Tang. approx

$$|e^{j\pi-0.1} - 1.1e^{j\pi}| \approx \sqrt{2}(0.1)$$



3. A two-pole band-pass filter has a gain of 1 at its center frequency. Its center frequency is $5\pi/8$ radians/sample and it has a bandwidth of 0.07 radians/sample.

- (1) (a) Find the approximate magnitude response of the band-pass filter at frequency 0 radians/sample.
- (4) (b) Draw a signal flow graph showing how the filter will be built. Annotate the graph making sure the coefficients are legible.



$$H(z) = \frac{b_0 z^2}{(z - 0.965e^{j\frac{5\pi}{8}})(z - 0.965e^{-j\frac{5\pi}{8}})}$$

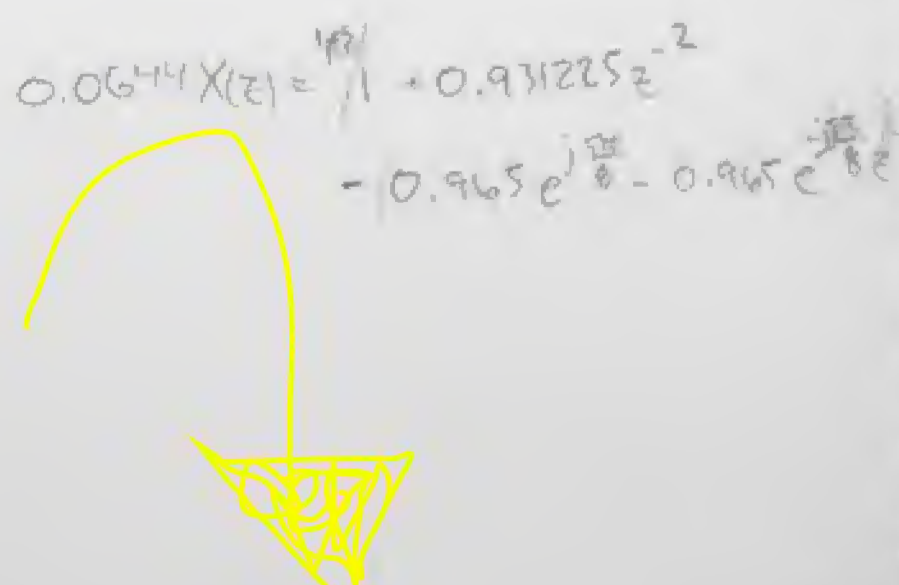
$$|H(e^{j\omega})| = \frac{b_0}{|e^{j\omega} - 0.965e^{j\frac{5\pi}{8}}| |e^{j\omega} - 0.965e^{-j\frac{5\pi}{8}}|} \Rightarrow |H(e^{j\frac{5\pi}{8}})| = 1 = \frac{b_0}{(0.035)(\frac{9.2cm}{5cm})}$$

$$|H(e^{j\omega})| = \frac{0.0644}{|e^{j\omega} - 0.965e^{j\frac{5\pi}{8}}| |e^{j\omega} - 0.965e^{-j\frac{5\pi}{8}}|}$$

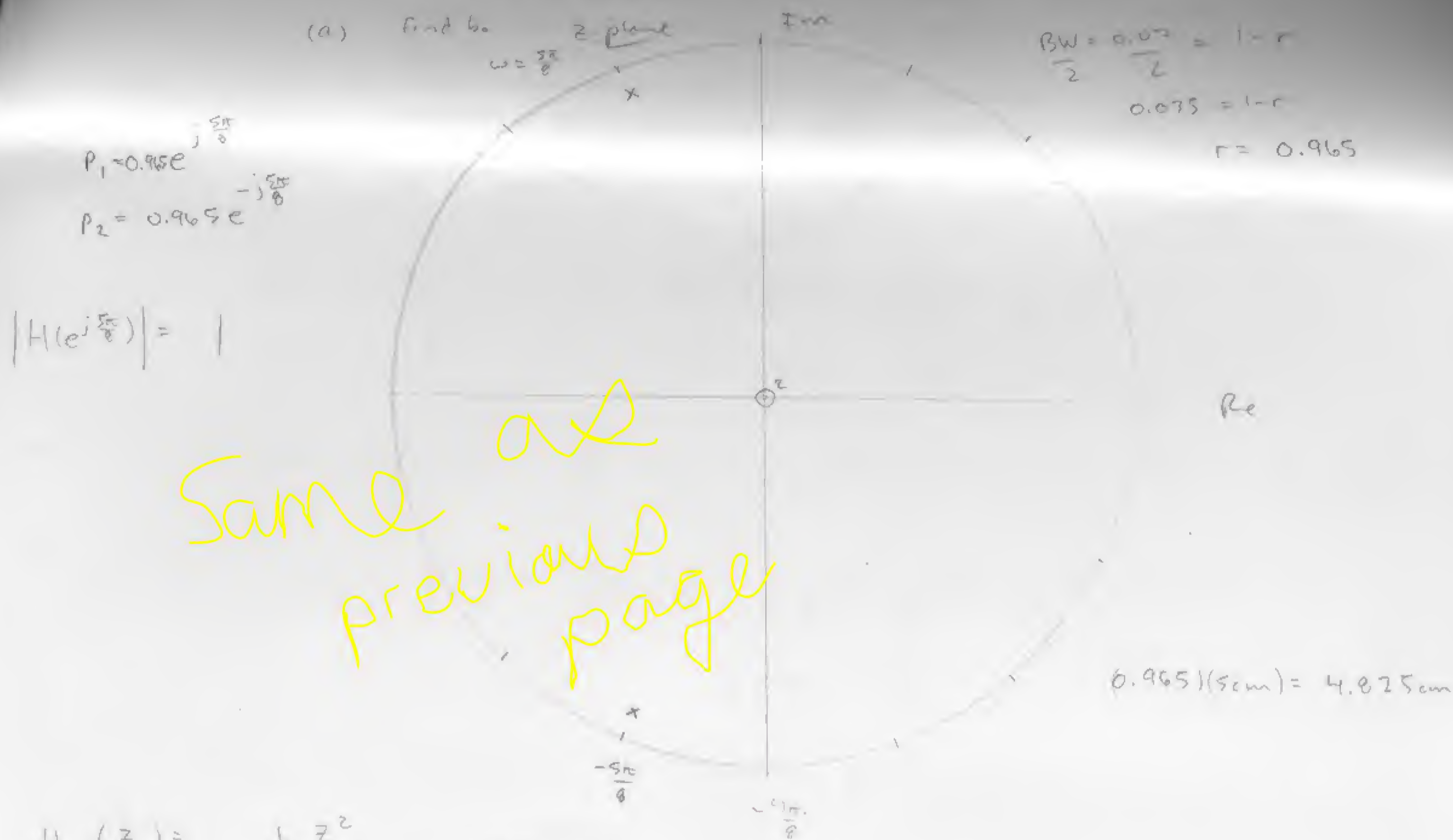
$$|H(e^{j0})| = \frac{0.0644}{(8.3cm \times \frac{1}{5cm})(8.3cm \times \frac{1}{5cm})} = \boxed{2.34 \times 10^{-2} = |H(e^{j0})|}$$

(b) $H(z) = \frac{z^2 \cdot 0.0644}{z^2 (1 - 0.965e^{j\frac{5\pi}{8}}z^{-1})(1 - 0.965e^{-j\frac{5\pi}{8}}z^{-1})} = \frac{Y(z)}{X(z)} \Rightarrow 0.0644X(z) = Y(z) - 0.93225z^{-2}Y(z) + 0.965e^{j\frac{5\pi}{8}}Y(z) - 0.965e^{-j\frac{5\pi}{8}}Y(z)$

$$0.0644X(z) = Y(z) [1 + 0.93225z^{-2} - 0.965e^{j\frac{5\pi}{8}}z^{-1} - 0.965e^{-j\frac{5\pi}{8}}z^{-1}]$$



Annotate the graph making sure the coefficients are legible.



$$H(z) = \frac{b_0 z^2}{(z - 0.965 e^{j\frac{5\pi}{8}})(z - 0.965 e^{-j\frac{5\pi}{8}})}$$

$$|H(e^{j\omega})| = \frac{1 b_0}{|e^{j\omega} - 0.965 e^{j\frac{5\pi}{8}}| |e^{j\omega} - 0.965 e^{-j\frac{5\pi}{8}}|} \Rightarrow |H(e^{j\frac{5\pi}{8}})| = 1 = \frac{b_0}{(0.075)(\frac{9.2cm}{5cm})}$$

$$b_0 = 0.0644$$

$$|H(e^{j\omega})| = \frac{0.0644}{|e^{j\omega} - 0.965 e^{j\frac{5\pi}{8}}| |e^{j\omega} - 0.965 e^{-j\frac{5\pi}{8}}|}$$

$$|H(e^{j0})| = \frac{0.0644}{(8.3cm + \frac{1}{5cm})(8.3cm + \frac{1}{5cm})} = \boxed{2.34 \times 10^{-2} = |H(e^{j0})|}$$

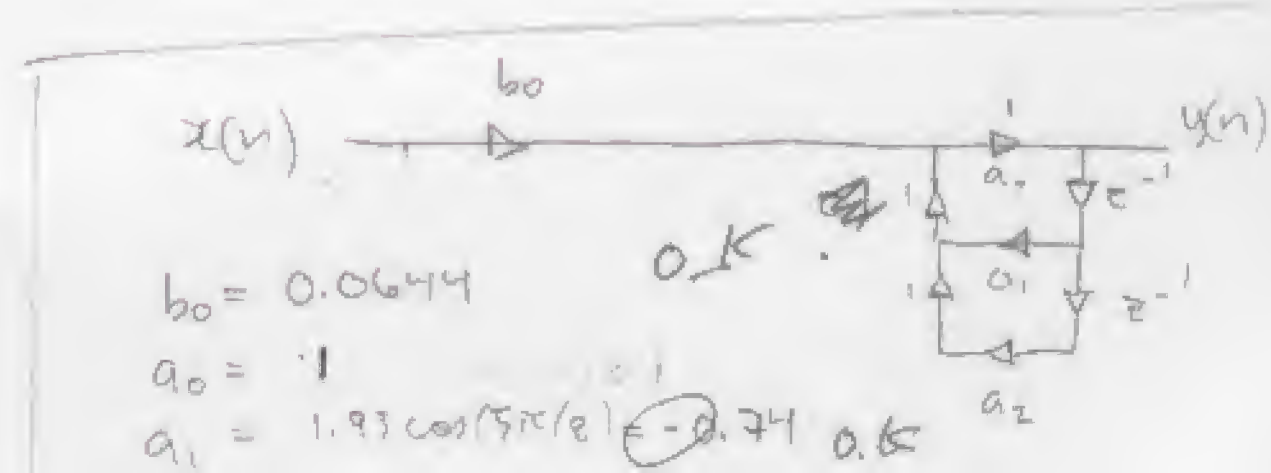
(b) $H(z) = \frac{z^2 \cdot 0.0644}{z^2 (1 - 0.965 e^{j\frac{5\pi}{8}} z^{-1})(1 - 0.965 e^{-j\frac{5\pi}{8}} z^{-1})} = \frac{Y(z)}{X(z)} \Rightarrow 0.0644 X(z) = \frac{Y(z)}{1 + 0.931225 z^{-2} - 0.965 z^{-1} 2 \cos \frac{5\pi}{8}}$

$$0.0644 X(z) = Y(z) [1 + 0.931225 z^{-2} - 0.965 z^{-1} 2 \cos \frac{5\pi}{8}]$$

$$0.0644 x(n) = y(n) + 0.931225 y(n-2) - 1.93 \cos \frac{5\pi}{8} y(n-1)$$

$$y(n) = 0.0644 x(n) - 0.931225 y(n-2) + 1.93 \cos \frac{5\pi}{8} y(n-1)$$

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4. A random sequence is generated using two coins, a penny and a dime. A sample is given a value of 2 if both coins show 'head', a value of -2 if both coins show 'tail' and a value of 0 if one coin shows 'head' and the other coin shows 'tail'. Only one coin is tossed on each new sample. The penny is tossed just before deciding even numbered samples and the dime is tossed just before deciding odd numbered samples.

- (2) (a) What is the DC component of the sequence? That is to say, what is the mean or average value of the sequence, which is often denoted μ or $\overline{x(n)}$?
- (3) (b) What is the average power per sample, i.e. average power of the sequence or mean square value, which is often denoted $\overline{x^2(n)}$?

(a)

	$x(n)$	
H, H,	= 2	$P_{HH} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$
H, T	= 0	$P_{HT} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{2}$
T, T	= -2	$P_{TT} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

DC component = $\sum_i x_i p_i = \frac{1}{4}(2) + 0\left(\frac{1}{2}\right) + \frac{1}{4}(-2) = \boxed{0 = \mu}$

(b) $\overline{x^2(n)} = \sum_i (x_i)^2 p_i = \frac{1}{4}(2)^2 + \frac{1}{2}(0)^2 + \frac{1}{4}(-2)^2 = 1 + 1$

= $\boxed{2 = \overline{x^2(n)}}$



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(5)

5. An NCO is used for a local oscillator in a frequency up-converter. The NCO runs at a nominal frequency of 44 MHz and must be within 1 Hz of 44 MHz. The oscillator must have a SNR greater than 60 dB. The sampling frequency is 200 Msamples/second. Find the length of the phase accumulator and the size of the ROM, i.e. find N_r , N_A and N_D .

$$f = 44 \text{ MHz} \pm 1 \text{ Hz} \quad \text{frequency resolution} = 2 \text{ Hz}$$

$$F_s = 200 \frac{\text{Msamples}}{\text{sec}}$$

$$\text{freq. res} = \frac{F_s}{2^{N_r}} \text{ Hz} \Rightarrow \frac{200 \text{ Msamples/sec}}{2^{N_r}} = 2$$

$$100 \text{ EG} = 2^{N_r}$$

$$\frac{\log(100 \text{ EG})}{\log 2} = N_r = 26.6 \text{ bits}$$

$$N_r = 27 \text{ bits}$$

check to decrease N_r :

$$44 \text{ MHz} = \frac{k F_s}{2^{N_r}}$$

$$\frac{44 \text{ MHz}}{200 \frac{\text{Msamples}}{\text{sec}}} \cdot 2^{27} = k = 29527900.16$$

$$\therefore = 29527900 \text{ can be divided by 4}$$

so last 2 bits of N_r are unused

$$\therefore \boxed{N_r = 25 \text{ bits}}$$

$$\text{SNR} = 60 \text{ dB} < -5.17 \text{ dB} + 6.02 N_A$$

$$N_A = 10.82 \Rightarrow 11 \text{ bits}$$

$$N_A = 11$$

$$N_D = 10 \text{ gives SNR} = 58.5 \text{ dB}$$

$$\text{SNR} = 60 \text{ dB} < 1.76 \text{ dB} + 6.02 N_D$$

$$N_D = 9.674 = 10 \text{ bits}$$

increase N_D

$$\text{try } N_D = 11 \text{ SNR} = 60.25 \text{ dB}$$

$$\text{ROM} = 2^{11} \text{ words} \times 11 \text{ bits/word}$$

$$\therefore \boxed{N_A = 11 \text{ bits}} \\ \boxed{N_D = 11 \text{ bits}}$$

- (3) 6. Find the Fourier series coefficients, c_k , for the periodic sequence given below. The period of the sequence is N , where N is odd.

$$x(n) = \begin{cases} a^n; & n = 0, 2, 4, \dots, N-1; \\ b^n; & n = 1, 3, 5, \dots, N-2; \end{cases}$$

- (2) 7. Find the Discrete Time Fourier Transform (DTFT) of the aperiodic sequence given below.

$$x(n) = \sin(\omega_0 n)(u(n) - u(n-M))$$

$$c_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi k}{N} n}$$

Always check bounds of summation

for n even

$$c_k = \frac{1}{N} \sum_{n=0}^{\frac{N-1}{2}} (a^2 e^{-j \frac{2\pi k}{N} n}) = \frac{1}{N} \left[\frac{1 - (a^2 e^{-j \frac{4\pi k}{N}})^{\frac{N-1}{2}+1}}{1 - a^2 e^{-j \frac{4\pi k}{N}}} \right]$$

for n odd

$$c_k = \frac{1}{N} \sum_{n=0}^{\frac{N-3}{2}} b^{(2n+1)} e^{-j \frac{2\pi k}{N} (2n+1)} = \frac{b e^{-j \frac{2\pi k}{N}}}{N} \left[\frac{1 - (b^2 e^{-j \frac{4\pi k}{N}})^{\frac{N-3}{2}+1}}{1 - b^2 e^{-j \frac{4\pi k}{N}}} \right]$$

$$= \frac{b e^{-j \frac{2\pi k}{N}}}{N} \left[\frac{(1 - (b^2 e^{-j \frac{4\pi k}{N}})^{\frac{N-1}{2}}) (1 - b^2 e^{-j \frac{4\pi k}{N}})}{(1 - b^2 e^{-j \frac{4\pi k}{N}}) (1 - b^2 e^{-j \frac{4\pi k}{N}})} \right]$$

$$= \frac{b e^{-j \frac{2\pi k}{N}}}{N} \left[\frac{(1 - (b^2 e^{-j \frac{4\pi k}{N}})^{\frac{N-1}{2}}) (1 - b^2 e^{-j \frac{4\pi k}{N}})}{1 + b^4 - 2b^2 \cos(\frac{4\pi k}{N})} \right]$$

$$c_k = \begin{cases} \frac{1}{N} \left[\frac{(1 - (a^2 e^{-j \frac{4\pi k}{N}})^{\frac{N-1}{2}}) (1 - a^2 e^{-j \frac{4\pi k}{N}})}{1 + a^4 - 2a^2 \cos(\frac{4\pi k}{N})} \right] & ; \text{ } n \text{ even} \\ \frac{b e^{-j \frac{2\pi k}{N}}}{N} \left[\frac{(1 - (b^2 e^{-j \frac{4\pi k}{N}})^{\frac{N-1}{2}}) (1 - b^2 e^{-j \frac{4\pi k}{N}})}{1 + b^4 - 2b^2 \cos(\frac{4\pi k}{N})} \right] & ; \text{ } n \text{ odd} \end{cases}$$

DTFT

$$x(n) = \sin \omega_0 n [u(n) - u(n-M)]$$



$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{M-1} \sin \omega_0 n e^{-j\omega n} = \frac{1}{2j} \left[\sum_{n=0}^{M-1} e^{j\omega_0 n} e^{-j\omega n} - \sum_{n=0}^{M-1} e^{-j\omega_0 n} e^{-j\omega n} \right]$$

$$= \frac{1}{2j} \left[\frac{1 - (e^{j\omega_0} e^{-j\omega})^M}{1 - e^{j\omega_0} e^{-j\omega}} - \frac{1 - (e^{-j\omega_0} e^{-j\omega})^M}{1 - e^{-j\omega_0} e^{-j\omega}} \right] \checkmark$$

$$= \frac{1}{2j} \left[\frac{(1 - e^{j(\omega_0 - \omega)M})(1 - e^{-j(\omega_0 + \omega)})}{1 - e^{j\omega_0} e^{-j\omega} - e^{j\omega_0} e^{-j\omega}} - \frac{(1 - e^{-j(\omega_0 + \omega)M})(1 - e^{j(\omega_0 - \omega)})}{1 - e^{-j\omega_0} e^{-j\omega} - e^{-j\omega_0} e^{-j\omega}} \right]$$

$$= \frac{1}{2j} \left[\frac{(1 - e^{j(\omega_0 - \omega)M})(1 - e^{-j(\omega_0 + \omega)}) - (1 - e^{-j(\omega_0 + \omega)M})(1 - e^{j(\omega_0 - \omega)})}{1 + e^{-2j\omega} - 2e^{j\omega} \cos \omega_0} \right]$$

@ $\omega = \omega_0$ we have $\frac{0}{0}$ for ① and @ $\omega = -\omega_0 \Rightarrow \frac{0}{0}$ for ②

for ① $\frac{1}{2j} \left[\frac{1 - e^{j\omega_0 M} e^{-j\omega M}}{1 - e^{j\omega_0} e^{-j\omega}} \right]$ L'hopital's rule

$$\rightarrow \frac{1}{2j} \left[\frac{1 + jM e^{j\omega_0 M} e^{-j\omega M}}{1 + j e^{j\omega_0} e^{-j\omega}} \right] \Rightarrow @ \omega = \omega_0 = \frac{1}{2j} \frac{1 + jM}{1 + j} \delta(\omega - \omega_0)$$

@ $\omega = -\omega_0$ we have $\frac{0}{0}$ for ②

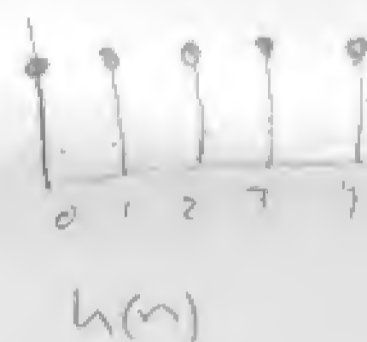
$$\cdot \frac{1}{2j} \left[\frac{1 - e^{-j\omega_0 M} e^{-j\omega M}}{1 - e^{-j\omega_0} e^{-j\omega}} \right] \Rightarrow \frac{d}{d\omega} = \frac{1}{2j} \left[\frac{1 + jM e^{-j\omega_0 M} e^{-j\omega M}}{1 + j e^{-j\omega_0} e^{-j\omega}} \right]$$

$$@ \omega = -\omega_0 = \frac{1}{2j} \left[\frac{1 + jM}{1 + j} \right] \delta(\omega + \omega_0)$$

8. A sequence, $x(n)$, is generated as follows: A value of +1 or -1 is assigned with probability 0.5 to samples $\dots x(-5), x(0), x(5), \dots$. The samples between these are filled with the value of the lower sample, for example $x(4) = x(3) = x(2) = x(1) = x(0)$.

- (2) (a) Find the autocorrelation function for the sequence $x(n)$.
 (3) (b) Find the power spectrum of the sequence $x(n)$.

(a) $x(n)$ model as filter and input $x'(n)$



$$\begin{aligned} x'(n+1) &= x'(n-1) \\ -x'(n+4) &= x'(n-3) \\ -x'(n+8) &= x'(n) \end{aligned}$$

$$x'(n) = \begin{cases} \pm 1 & x(n5); n \text{ integer} \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} r_{x'x'}(0) &= \sum_i x_i^2 p_i = \frac{1}{5} \left(\frac{1}{2} \right) (1)^2 + \frac{1}{5} \left(\frac{1}{2} \right) (-1)^2 \\ &= \frac{1}{5} \end{aligned}$$

$$r_{x'x'}(m) = \frac{1}{5} \delta(m)$$

$$S_{x'x'}(e^{j\omega}) = 1/5$$

$$x'(n) + x'(n-1) + x'(n-2) + x'(n-3) + x'(n-4) = x(n)$$

$$h(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \delta(n-4)$$

$$H(e^{j\omega}) = 1 + e^{-j\omega} + e^{-2j\omega} + e^{-3j\omega} + e^{-4j\omega}$$

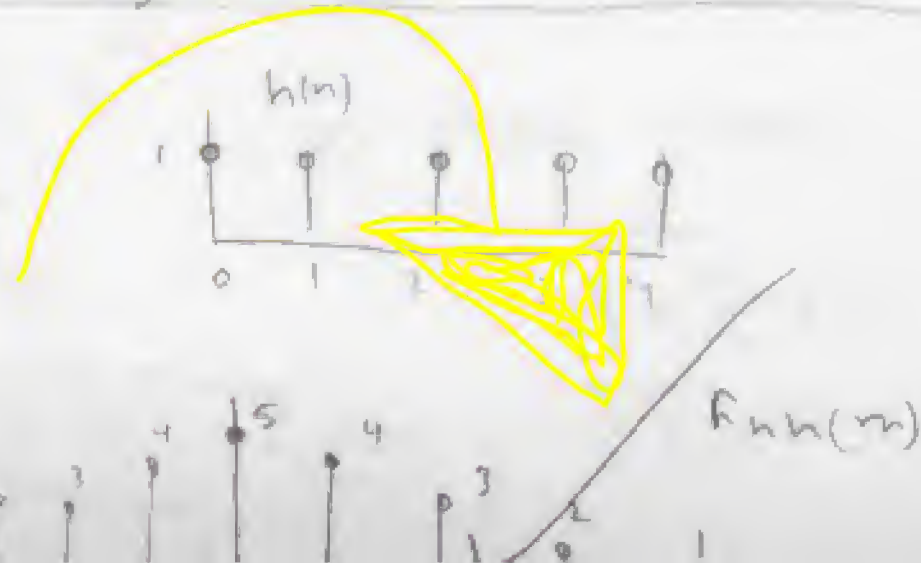
$$= e^{-2j\omega} (e^{2j\omega} + e^{-2j\omega} + e^{j\omega} + e^{-j\omega} + 1)$$

$$= e^{-2j\omega} (2 \cos(2\omega) + 2 \cos \omega + 1)$$

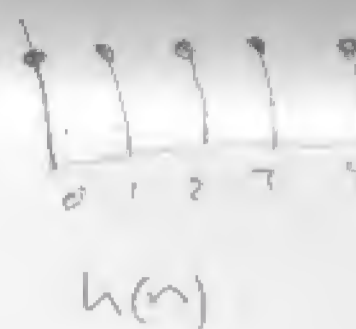
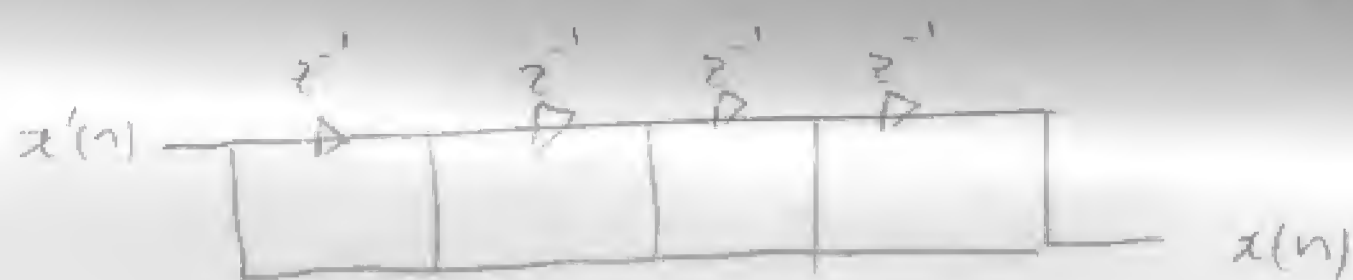
$$|H(e^{j\omega})| = (1 + 2 \cos \omega + 2 \cos 2\omega)$$

$$(b) S_{xx}(e^{j\omega}) = |H(e^{j\omega})|^2 S_{x'x'}(e^{j\omega}) = \frac{1}{5} (1 + 2 \cos \omega + 2 \cos 2\omega)^2$$

$$r_{hh}(m) = \sum_{n=-\infty}^{\infty} x(n) x(n-m)$$



(3)

(b) Find the power spectrum of the sequence $x(n]$.(a) $x[n]$ model as filter and input $x'(n)$ 

$$\begin{aligned} x'(n) &= x[n-1] \\ -x'(n-2) &= x'[n-3] \\ -x'(n-4) &= x[n] \end{aligned}$$

$$x'(n) = \begin{cases} \pm 1 & x[n], n \text{ integer} \\ 0 & \text{otherwise} \end{cases}$$

~~Solve~~ $\sum_i x_i^2 p_i = \frac{1}{5} \left(\frac{1}{2} \right) (1)^2 + \frac{1}{5} \left(\frac{1}{2} \right) (-1)^2$

as previous page

$$r_{x'x'}(m) = \frac{1}{5} \delta(m)$$

$$S_{x'x'}(e^{j\omega}) = 1/5$$

$$x'(n) + x'(n-1) + x'(n-2) + x'(n-3) + x'(n-4) = x[n]$$

$$h(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \delta(n-4)$$

$$H(e^{j\omega}) = 1 + e^{-j\omega} + e^{-2j\omega} + e^{-3j\omega} + e^{-4j\omega}$$

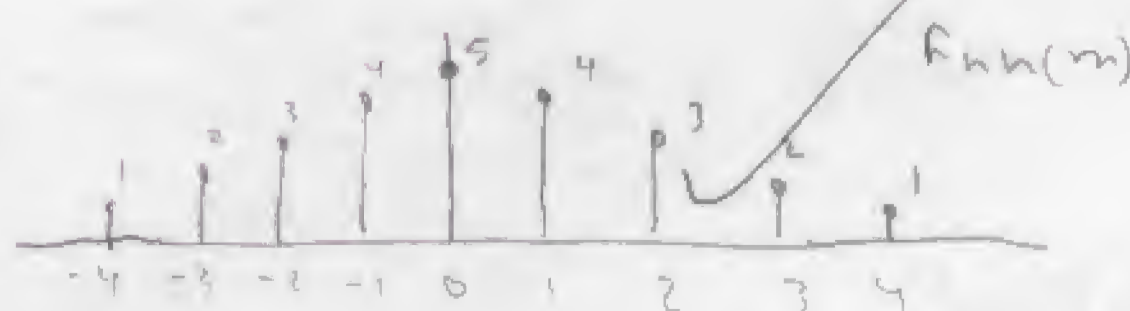
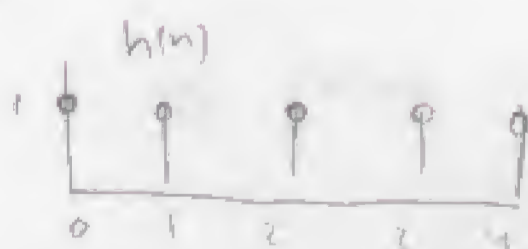
$$= e^{-2j\omega} (e^{2j\omega} + e^{-2j\omega} + e^{j\omega} + e^{-j\omega} + 1)$$

$$= e^{-2j\omega} (2 \cos(2\omega) + 2 \cos \omega + 1)$$

$$|H(e^{j\omega})| = (1 + 2 \cos \omega + 2 \cos 2\omega)$$

$$(b) S_{xx}(e^{j\omega}) = |H(e^{j\omega})|^2 S_{x'x'}(e^{j\omega}) = \frac{1}{5} (1 + 2 \cos \omega + 2 \cos 2\omega)^2$$

$$r_{hh}(m) = \sum_{n=-\infty}^{\infty} h(n) h(n-m)$$



$$r_{xx}(m) = r_{hh}(m) \otimes r_{x'x'}(m)$$

$$\begin{aligned} r_{xx}(m) &= \delta(m) + \frac{4}{5} \delta(m-1) + \frac{4}{5} \delta(m+1) + \frac{3}{5} \delta(m-2) + \frac{3}{5} \delta(m+2) + \frac{2}{5} \delta(m-3) + \frac{2}{5} \delta(m-3) \\ &+ \frac{1}{5} \delta(m-4) + \frac{1}{5} \delta(m+4) \end{aligned}$$

(5)

9. An NCO has an accumulator with 12 bits and a ROM that is 256 words by 8 bits per word. What is the SNR of the output sinusoid if the output frequency is 1616/4096 cycles per sample?

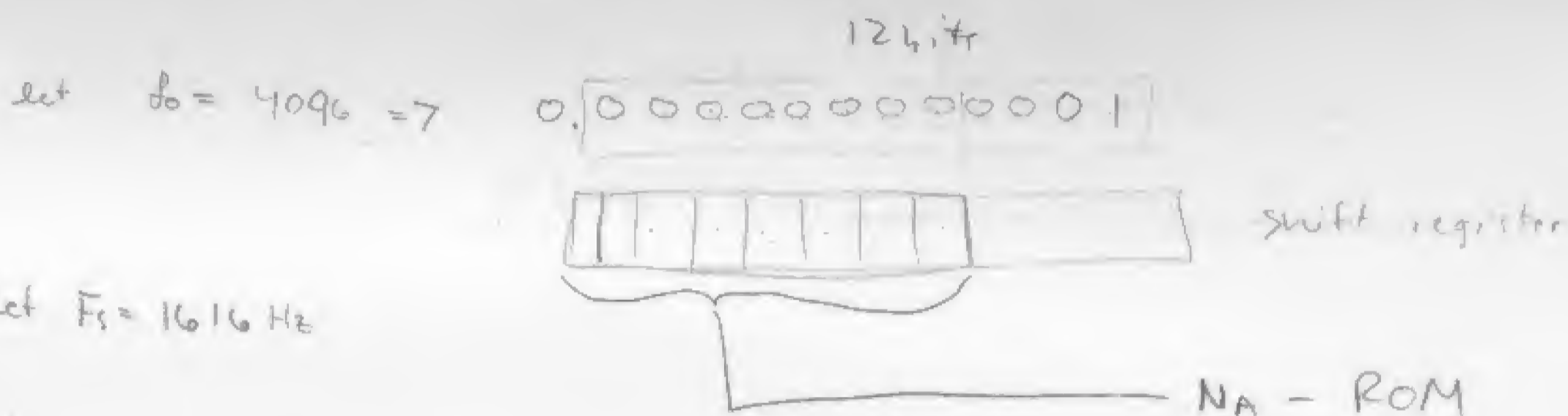
$$N_r = 12$$

$$N_A = 8$$

$$N_o = 8$$

$$\frac{A^2}{2} = \frac{(2^{N_o} - 1)^2}{2} = \frac{2^{2N_o}}{8}$$

$$SNR = \frac{\text{SIGNAL POWER}}{\overline{q_a^2(n)} + \overline{q_d^2(n)}}$$



$$\therefore N_A = N_r + 4 = 7 \text{ worst case}$$

$$\therefore \overline{q_a^2(n)} = \frac{A^2}{2} \left(\frac{S^2}{12} \right)$$

where $S = \frac{2\pi}{2^{N_A}}$

This is probably not right but

assume worst case

$$SNR = \frac{2^{2N_o}}{\frac{\pi^2}{3} 2^{2(N_o - N_A)} + \frac{2}{3}} = 42.19 \text{ dB}$$

✓ This may be on the right track

$$\frac{1616 \text{ cycles}}{4096 \text{ samples}} = \frac{k F_s}{2^{12}} \therefore k F_s = 1616$$

~~1616~~

$F_s = 101$

$k = 1616$

1600100000000

$$f_0 = \frac{k}{2^{N_r}} \Rightarrow k = \frac{1616 \cdot 2^{12}}{4096}$$

$$= 1616$$

k divisible enough to have no phase error

no phase error \therefore

$$\overline{q_a^2(n)} = 0$$

$$\overline{q_d^2(n)} = 1/12$$

so

SNR

$$= \frac{(2^{2N_o} - 1)^2}{2}$$

$$= 49.8 \text{ dB}$$

$$N_s = 12$$

$$N_r = 8$$

$$N_o = 8$$

$$\frac{1^2}{2} = \frac{1^{2N_o} - 1}{2} = \frac{2^{2N_o} - 1}{8}$$

$$SNR = \frac{\text{SIGNAL POWER}}{\overline{q_a^2(n)} + \overline{q_d^2(n)}}$$

let $f_o = 4096 \Rightarrow 0.0000000000000001$ 12 bits

let $F_s = 1616 \text{ Hz}$



$\therefore N_A = N_r + 4 \Rightarrow$ worst case

$\therefore \overline{q_a^2(n)} = \frac{A^2}{2} \left(\frac{S^2}{12} \right)$

where $S = \frac{2\pi}{N_A} N_o$

Same as previous page

This is probably not right had

assume worst case

$$SNR = \frac{2^{2N_o}}{\frac{\pi^2}{3} 2^{2(N_o - N_A)} + \frac{2}{3}} = 42.19 \text{ dB}$$

This may be on the right track

$\frac{1616 \text{ cycles}}{4096 \text{ samples}} = \frac{k F_s}{2^{12}} \therefore k F_s = 1616$

~~1616~~ $F_s = 101$ $k = 1616$ 2^{12}

1000100000000000

$f_o = \frac{k}{2^{N_r}} \Rightarrow k = \frac{1616}{4096} \cdot 2^{12}$
 $= 1616$

no phase error \therefore

$\overline{q_a^2(n)} = 0$
 $\overline{q_d^2(n)} = 1/12$

so SNR $\left\{ \begin{array}{l} k \text{ divisible enough} \\ \text{to have no phase error} \end{array} \right\}$

$$= \frac{(2^{2N_o - 1})^2 / 2}{1/2} = 49.8 \text{ dB}$$